Interval Observer Approach to Output Stabilization of Time-Varying Input Delay Systems

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Abstract— **The output stabilization problem for a linear system with an unknown time-varying bounded input delay is considered. The interval observer technique is applied in order to obtain guaranteed interval estimates of the system state. The procedure of the interval observer design, which is based on resolving of the Sylvester's equation, is discussed. Interval predictor is introduced and applied for a linear output stabilizing feedback design. The design procedure is based on Linear Matrix Inequalities (LMI). The theoretical results are supported by numerical simulations.**

I. Introduction and related works

A time-varying input delay arises in models of control systems due to several reasons. Usually its presence is motivated by a physical nature of a control plant. It may be involved by transport delays (like in chemical or pneumatic systems) or computational delay (e.g. in digital controllers or communication networks [16]). Time-varying input delay can be introduced "artificially" in order to model a sampling effect (see, for example, [25], [11], [10]).

Control of a system with input delay is an important problem treated in the literature (see, for example, [22], [15], [18] and references within). The predictor-based feedback [24] is a very common tool for control design if the delay is known. This method is well-developed for both constant and time-varying delay cases [1], [27], [15]. It has been effectively used even for nonlinear [3] and sliding mode control systems [20]. If delay is constant, but unknown, then delay estimation technique [4] and/or the delay-adaptive control approach [5] can be applied. This approach is also implicitly based on prediction technique. For *unknown input delay* the predictor-based feedback design has to be accompanied with robustness analysis [15], [28].

Typically, the predictor-based approach is effectively applicable if the whole state-vector of a system is measured [24], [22], [28], [15], [20]. The observer design for systems with time-varying input and state delay is presented in [23]. The results related to designing of an *output* predictor feedback for systems with input delays, which are known and constant, can also be found, for example, in [26] and [14]. The adaptive output feedback regulator for a chain of integrators with an unknown time-varying delay in the input is presented in [6].

The present paper uses a recently developed technique of interval observers [12], [17], [21], [19] in order to tackle the problem of the output-based control design for linear multiple input and multiple output systems with unknown, bounded but time-varying input delay. For the system without delays, an interval observer provides the guaranteed interval estimates of the system state in a real-time. This property helps in controlling the transition processes with respect to system state [7]. This paper extends the interval observer technique to systems with unknown time-varying input delay and presents the interval prediction scheme (interval predictor), that allows us to realize a predictor-based feedback design using the LMI technique.

The paper is organized as follows. The next section describes notations to be used in the paper. The section 3 presents the problem statement and basic assumptions. After that, the interval observer and interval predictor are introduced. The control design algorithm is given in the section 5. Finally, a numerical example and conclusions are presented.

II. Notations

- The set of real numbers is denoted by $\mathbb{R}$.
- The set of Hurwitz matrices from $\mathbb{R}^{n\times n}$ is denoted by $\mathcal{H}$.
- The set of Metzler matrices from $\mathbb{R}^{n\times n}$ is denoted by $\mathcal{M}$, i.e.

$$R=\{r_{ij}\}_{i,j=1}^{n}\in\mathcal{M}\quad\Leftrightarrow\quad r_{ij}\geq 0 \text{ for } i\neq j.$$

- The inequality $F\succ 0$ ($F\prec 0$) for a symmetric matrix $F\in\mathbb{R}^{n\times n}$ is meant positive(negative) definiteness of the matrix F. The order relations $F\succeq 0$ and $F\preceq 0$ are used in order to assign the positive and negative semidefiniteness of the matrix F, respectively.
- If $P\in\mathbb{R}^{n\times n}$ then $\lambda_{\max}(P)$ is the maximum eigenvalue of P.
- The inequalities $x>y$, $x<y$, $x\geq y$ and $x\leq y$ written for vectors $x,y\in\mathbb{R}^n$ are to be understood in a componentwise sense.
- The identity matrix of the size $n\times n$ is denoted by I_n; the square zero matrix of the size $n\times n$ is denoted by

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0_n; the rectangular zero matrix of the size $n \times m$ is denoted by $0_{n\times m}$.

III. PROBLEM STATEMENT

Consider the input delay control system of the form

$$\dot{x} = Ax + Bu(t-h(t)), \quad y = Cx, \tag{1}$$

where $x \in \mathbb{R}^n$ is the system state, $u \in \mathbb{R}^m$ is the vector of control inputs, $y \in \mathbb{R}^k$ is the measured output, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$ and $C \in \mathbb{R}^{k\times n}$ are known matrices and the time-varying input delay $h(t)$ is assumed to be locally Lebesgue measurable and unknown but within the bounded interval:

$$0 \le \underline{h} \le h(t) \le \overline{h}, \tag{2}$$

where the numbers $\underline{h}$ and $\overline{h}$ are given.

The system (1) is studied with the initial conditions:

$$\begin{array}{c} x(0) = x_0, \\ u(t) = v(t) \text{ for } t \in [-\overline{h}, 0), \end{array} \tag{3}$$

where $v(t)$ is some continuous function.

Assumption 1: The pair (A, B) is controllable and the pair (A, C) is observable.

Assumption 2: The information on the control signal $u(t)$ on the time interval $[t - \overline{h}, t)$ can be stored and used for control design purposes.

Assumption 3: The set $\Omega \subset \mathbb{R}^n$ of admissible initial conditions $x_0 \in \Omega$ of the system (1) is assumed to be **bounded** and **known**.

Remark that the second assumption is usual for a predictor-based approach to control design.

The main objective of this paper is to design a control algorithm for exponential stabilization of the system (1), i.e. for some numbers $c, r > 0$ any solution of the closed-loop system (1) has to satisfy the inequality

$$\|x(t)\| \le ce^{-rt}, \qquad t > 0$$

if $x(0) \in \Omega$.

IV. INTERVAL OBSERVER AND INTERVAL PREDICTOR DESIGN

A. Concept of the interval observation

In order to explain the key idea of the interval observation [12], [21] used below let us consider initially the following delay free system:

$$\dot{x} = Ax + f(t), \qquad y = Cx,$$

where x, y, A, C have the same sense as in the system (1) and the unknown function $f(t) \in \mathbb{R}^n$ is assumed to be componentwise bounded as follows

$$\underline{f}(t) \le f(t) \le \overline{f}(t) \quad \text{ for } \quad t > 0,$$

where the functions $\underline{f}$ and $\overline{f}$ are known.

Let there exists a matrix $L \in \mathbb{R}^{n\times k}$ such that $A + LC \in \mathcal{H} \bigcap \mathcal{M}$. Then the observation system

$$\left\{ \begin{array}{l} \dot{\overline{x}} = A\overline{x} + \overline{f}(t) + L(C\overline{x} - y), \\ \dot{\underline{x}} = A\underline{x} + \underline{f}(t) + L(C\underline{x} - y). \end{array} \right.$$

has the following property:

$$\underline{x}(0) \le x(0) \le \overline{x}(0) \quad \Rightarrow \quad \underline{x}(t) \le x(t) \le \overline{x}(t), \quad t > 0.$$

Indeed, consider the error equations for $\overline{e} = \overline{x} - x$ and $\underline{e} = x - \underline{x}$:

$$\left\{ \begin{array}{l} \dot{\overline{e}} = (A + LC)\overline{e} + \overline{f}(t) - f(t), \\ \dot{\underline{e}} = (A + LC)\underline{e} + f(t) - \underline{f}(t). \end{array} \right.$$

Since $A + LC \in \mathcal{M}$ and $\overline{f}(t) - f(t) \ge 0$, $f(t) - \underline{f}(t) \ge 0$, then the last system is positive [8], i.e. $\overline{e}(0) \ge 0$ and $\underline{e}(0) \ge 0$ implies $\overline{e}(t) \ge 0$ and $\underline{e}(t) \ge 0$ for $t > 0$. Moreover, the system of error equations is also stable $A+LC \in \mathcal{H}$. Hence, if $\overline{f}(t) - f(t) \to 0$ and $\underline{f}(t) - f(t) \to 0$ as $t \to +\infty$ then $\overline{e}(t) \to 0$ and $\underline{e}(t) \to 0$ as $t \to +\infty$.

Therefore, the interval observation technique allows us to provide the guaranteed estimation of the state for the system with unknown input. In our case the input $u(t-h(t))$ of the system (1) is unknown, since $h(t)$ is unknown. However, according to Assumption 2 it can be estimated. So, the interval observer can be designed for the system (1).

B. Interval observer for an input delay system

Let us introduce the following notations

$$\Delta u(\tau, \theta) := u(\tau - \theta) - u(\tau), \tag{4}$$

$$\underline{\Delta} B'u(\tau) := \min_{\theta \in [0, \overline{h} - \underline{h}]} B'\Delta u(\tau, \theta), \tag{5}$$

$$\overline{\Delta} B'u(\tau) := \max_{\theta \in [0, \overline{h} - \underline{h}]} B'\Delta u(\tau, \theta), \tag{6}$$

where $\min(\max)$ is considered componentwise and B' is some matrix of an appropriate dimension.

Lemma 4: Under Assumptions 1-3 there always exist matrices $L \in \mathbb{R}^{n\times k}$ and $S \in \mathbb{R}^{n\times n}, \det(S) \neq 0$ such that

$$A + LC \in \mathcal{H}, \quad S^{-1}(A + LC)S \in \mathcal{M}, \tag{7}$$

and the interval observer of the form

$$\begin{array}{c} \dot{\underline{x}}(t)=\tilde{A}\underline{x}+\tilde{B}u(t-\underline{h})+\underline{\Delta}\tilde{B}u(t-\underline{h})+\tilde{L}(\tilde{C}\underline{x}-y), \\ \dot{\overline{x}}(t)=\tilde{A}\overline{x}+\tilde{B}u(t-\underline{h})+\overline{\Delta}\tilde{B}u(t-\underline{h})+\tilde{L}(\tilde{C}\overline{x}-y), \\ \underline{x}(0) \le \tilde{x}(0) \le \overline{x}(0), \\ \tilde{A} = S^{-1}AS, \tilde{B} = S^{-1}B, \tilde{L} = S^{-1}L, \tilde{C} = CS, \\ \tilde{x} = S^{-1}x, \end{array} \tag{8}$$

guarantees

$$\underline{x}(t) \le \tilde{x}(t) \le \overline{x}(t) \quad \forall t > 0, \tag{9}$$

and $\underline{x}(t) \to \tilde{x}(t)$, $\overline{x}(t) \to \tilde{x}(t)$ if $u(t) \to 0$ for $t \to +\infty$.

Sketch of the proof.

I. Since the pair (A, C) is observable then an appropriate selection of the matrix L can assign any real disjoint negative spectrum of the matrix $A + LC$. Then the matrix S can be defined as Jordan transformation for $A + LC$, which is real in this case.

II. The system ordinary differential equations, which describes an evolution of the observation errors $\underline{e} = \tilde{x} - \underline{x}$ and $\overline{e} = \overline{x} - \tilde{x}$, is positive [8] and stable. So, the inequalities (9) hold. ■

Remark 5: To realize in practice the interval observer (8) the condition $\underline{x}(0) \leq \tilde{x}(0) \leq \overline{x}(0)$ must be guaranteed. Since the set of admissible initial conditions Ω is assumed to be known, the required inequality can be ensured. For example, if $\Omega = \{x \in \mathbb{R}^n : x^T P x < 1\}, P \succ 0$, then $\tilde{x}^T S^T P S \tilde{x} < 1$ and $\underline{x}_i(0) = -\overline{x}_i(0) = -1/\lambda_{\min}(S^T P S),\ i = 1, 2, ..., n$. Some similar estimates can be also presented if Ω is some polyhedron.

For practical reasons it is important to design interval observer with predefined Metzler matrix. Let some Hurwitz and Metzler matrix R be given and suppose we need to find S and L such that

$$S^{-1}(A + LC)S = R.$$

Denote $X = S^{-1}$ and $Y = S^{-1}L$. In this case the required equality can be rewritten in the form of Sylvester's equation [2]

$$XA + YC = RX, \tag{10}$$

where $X \in \mathbb{R}^{n\times n}$ and $Y \in \mathbb{R}^{n\times k}$.

Proposition 6: [2], [21] If the matrix R has disjoint spectrum and the pair (A, C) is observable then the equation (10) has a solution.

Remark that R may have complex eigenvalues.

Equation (10) can be rewritten in the form of the system of linear algebraic equations

$$Wz = 0, \tag{11}$$

where

$$W = \left(\begin{array}{cc} I_n \otimes A^T - R \otimes I_n & I \otimes C' \end{array}\right),$$

$$z = (x_{11}, x_{21}, ... , x_{n1}, x_{21}, ..., x_{nn}, y_{11}, ..., y_{n1}, y_{21}, ..., y_{nk})^T,$$

where $\otimes$ is the Kroneker product. So, numerically the required solution of the equation (10) can be found as an element of the null space of the matrix W.

C. Interval Predictor

Consider the system (8). By analogy with Artstein transformation [1] let us introduce the following predictor variables:

$$\underline{z}(t) = e^{\tilde{A}\underline{h}}\underline{x}(t) + \int_{-\underline{h}}^{0} e^{-A\theta}\left(\tilde{B}u(t+\theta) + \underline{\Delta}\tilde{B}u(t+\theta)\right)d\theta, \tag{12}$$

$$\overline{z}(t) = e^{\tilde{A}\underline{h}}\overline{x}(t) + \int_{-\underline{h}}^{0} e^{-A\theta}\left(\tilde{B}u(t+\theta) + \overline{\Delta}\tilde{B}u(t+\theta)\right)d\theta, \tag{13}$$

which are correctly defined due to Assumption 2. The variable $\underline{z}(t)$ and $\overline{z}(t)$ estimates the future values of the observed states $\underline{x}(t+\underline{h})$ and $\overline{x}(t+\underline{h})$, respectively. Obviously that for $\underline{h} = 0$ there is nothing to predict and $\underline{z}(t) = \underline{x}(t)$, $\overline{z}(t) = \overline{x}(t)$. It is easy to check that the predictor equations have the form

$$\begin{array}{l} \dot{\underline{z}}(t) = \tilde{A}\underline{z}(t) + \tilde{B}u(t) + \underline{\Delta}\tilde{B}u(t) - e^{\tilde{A}\underline{h}}\tilde{L}\tilde{C}\underline{e}(t), \\ \dot{\overline{z}}(t) = \tilde{A}\overline{z}(t) + \tilde{B}u(t) + \overline{\Delta}\tilde{B}u(t) + e^{\tilde{A}\underline{h}}\tilde{L}\tilde{C}\overline{e}(t). \end{array} \tag{14}$$

Below it is shown that a stabilizing control for the original system can be designed as a linear feedback with respect to predictor variables.

V. Stabilizing control design

Assume that some interval observer (8) for the system (1) is designed, the matrices S, L are obtained by mean of resolving Sylvester's equation (10) and the matrices $\tilde{A}, \tilde{L}, \tilde{C}, \tilde{B}$ are computed by the formula (8).

Let us define the control in the form

$$u(t) = Kz(t), \quad z(t) = \frac{\overline{z}(t) + \underline{z}(t)}{2}, \tag{15}$$

where $K \in \mathbb{R}^{m\times n}$.

Remark 7: Let us mention that the control function can be selected in a more general form

$$u(t) = \underline{K}\,\underline{z}(t) + \overline{K}\overline{z}(t),$$

where $\underline{K}, \overline{K} \in \mathbb{R}^{m\times n}$. In particular, the gain matrices can be defined as $\underline{K} = \mu K$ and $\overline{K} = (1-\mu)K$, where $\mu \in [0, 1]$. This form of control implies some small changes in formulation and proof of the main theorem given below. We select $\mu = 0.5$ for simplicity and shortness. Moreover, such selection had allowed us to attain the best convergence rate during numerical simulations.

Let $\tilde{B}_i \in \mathbb{R}^{n\times m}, i = 1, 2, ..., n$ be the matrix such that i-th row of $\tilde{B}_i$ coincides with i-th row of the matrix $\tilde{B}$ but all other rows of $\tilde{B}_i$ are zero. Denote also $\tilde{B}_{n+i} = \tilde{B}_i, i = 1, 2..., n$.

Theorem 8: If for some given $\alpha, \beta, \gamma \in \mathbb{R}_+$ the matrices $X, Z, R_i, S_i \in \mathbb{R}^{n\times n},\ i = 1, 2, ..., 2n$ and the matrix $Y \in \mathbb{R}^{m\times n}$ satisfy the following LMI system

$$\begin{pmatrix} W_e & W_{ez} \\ W_{ez}^T & W_z \end{pmatrix} \preceq 0, X \succ 0, Z \succ 0, R_i \succ 0, S_i \succ 0, \tag{16}$$

where

$$W_e = \begin{pmatrix} \Pi_1 & \tilde{B}_1 Y & ... & \tilde{B}_{2n} Y \\ Y^T \tilde{B}_1^T & -e^{-\beta\Delta h} S_1 & ... & 0 \\ ... & ... & ... & ... \\ Y^T \tilde{B}_{2n}^T & ... & ... & -e^{-\beta\Delta h} S_{2n} \end{pmatrix},$$

$$W_z = \begin{pmatrix} \Pi_2 & \Pi_4 & \tilde{B}_1 Y & ... & \tilde{B}_{2n} Y \\ \Pi_4^T & \Pi_3 & \tilde{B}_1 Y & ... & \tilde{B}_{2n} Y \\ Y^T \tilde{B}_1^T & Y^T \tilde{B}_1^T & -e^{-\alpha\Delta h} R_1 & ... & 0 \\ ... & ... & ... & ... & ... \\ Y^T \tilde{B}_{2n}^T & Y^T \tilde{B}_{2n}^T & 0 & ... & -e^{-\alpha\Delta h} R_{2n} \end{pmatrix}$$

$$W_{ez} = \begin{pmatrix} Z\tilde{C}^T\tilde{L}^T e^{\tilde{A}^T\underline{h}} \left[\begin{array}{cc} I_n & I_n \end{array}\right] & 0 \\ 0 & 0 \end{pmatrix},$$

$$\Pi_1 = (\tilde{A} + \tilde{L}\tilde{C})Z + Z(\tilde{A} + \tilde{L}\tilde{C})^T + \beta Z,$$

$$\Pi_2 = \tilde{A}X + \tilde{B}Y + X\tilde{A}^T + Y^T\tilde{B}^T + \alpha X,$$

$$\Pi_3 = \tfrac{1}{4}\sum_{i=1}^{2n}(R_i + e^{\gamma\underline{h}}S_i) - \tfrac{2}{\Delta h}X,\ \Delta h := \overline{h} - \underline{h},$$

$$\Pi_4 = X\tilde{A}^T + Y^T\tilde{B}^T$$

then the system (1) together with the control (15) for

$$K = YX^{-1}$$

is exponentially stable with the convergence rate : $r \geq \min\{\alpha, \beta, \gamma\}$.

Sketch of the proof. I. Introducing the auxiliary state vector $e = \frac{\overline{e}-\underline{e}}{2}$ the closed-loop system (1), (8), (15) can be rewritten in the form

$$\begin{aligned}\dot{e}=&(\tilde{A}+\tilde{L}\tilde{C})e(t) - \tfrac{1}{2}\sum_{i=1}^{2n}\tilde{B}_iK\int\limits_{t-\underline{h}-\theta_i(t-\underline{h})}^{t-h(t)}\dot{z}(s)ds,\\ \dot{z}=&(\tilde{A}+\tilde{B}K)z(t) - \tfrac{1}{2}\sum_{i=1}^{2n}\tilde{B}_iK\int\limits_{t-\theta_i(t)}^{t}\dot{z}(s)ds+e^{\tilde{A}\underline{h}}\tilde{L}\tilde{C}e(t),\end{aligned} \tag{17}$$

where $\theta_i : \mathbb{R}_+ \to [0, \Delta h]$, $i = 1, 2, ..., 2n$ are some functions implicitly depended on $z(\cdot)$ and $e(\cdot)$.

II. In order to analyze a stability of (17) the following Lyapunov-Krasovskii functional is used:

$$V(t, e(t), z(t), \dot{z}(\cdot)) = V_e + V_z + V_{ez},$$

$$V_e(t, e(t), \dot{z}(\cdot))=e^T(t)Qe(t)+$$

$$\Delta h\sum_{i=1}^{2n}\int\limits_{-\Delta h}^{0}\int\limits_{t-\underline{h}+\theta}^{t-\underline{h}}\frac{e^{\beta(s-t+\underline{h})}}{4}\dot{z}^T(s)PS_iP\dot{z}(s)ds\, d\theta,$$

$$V_z(t, z(t), \dot{z}(\cdot)) = z^T(t)Pz(t)+$$

$$\Delta h\sum_{i=1}^{2n}\int\limits_{-\Delta h}^{0}\int\limits_{t+\theta}^{t}\frac{e^{\alpha(s-t)}}{4}\dot{z}^T(s)PR_iP\dot{z}(s)ds\, d\theta,$$

$$V_{ez}(\dot{z}(\cdot))=(\Delta h)^2\sum_{i=1}^{2n}\int\limits_{t-\underline{h}}^{t}\frac{e^{\gamma(s-t+\underline{h})}}{4}\dot{z}^T(s)PS_iP\dot{z}(s)ds,$$

where $\alpha, \beta, \gamma \in \mathbb{R}_+,\ P, Q, S_i, R_i \in \mathbb{R}^{n\times n}, P \succ 0, Q \succ 0, S_i \succ 0, R_i \succ 0, i = 1, 2, ..., 2n.$

The structure of term V_z of the LKF V is similar to the one from the paper [9], [13]. The terms V_{ez} and V_e are motivated by the extended system (17), that implicitly contains both system state and observer state.

The LMI (16) for $Z = Q^{-1}$ and $P = X^{-1}$ implies that the functional V is exponentially decreasing to zero along the trajectories of the system (17).

■

It can be shown that under conditions of controllability of the pair $\{A, B\}$ and observability of the pair $\{A, C\}$ the LMI system (16) is feasible at least for sufficiently small Δh. Indeed, for $\Delta h \to 0$ it is easy to see that feasibility of the LMI (16) follows from $\Pi_1 < 0$ and $\Pi_2 < 0$. The last inequalities are feasible under Assumption 1.

VI. Numerical examples

A. Linear oscillator

Consider the system (1) with $n = 2, k = m = 1$ and parameters

$$A = \begin{pmatrix} 0 & 1\\ -1 & 0\end{pmatrix}, B = \begin{pmatrix} 0\\ 1\end{pmatrix}, C = \begin{pmatrix} 1 & 0\end{pmatrix}.$$

and $\underline{h} = 1$, $\overline{h} = 2$.

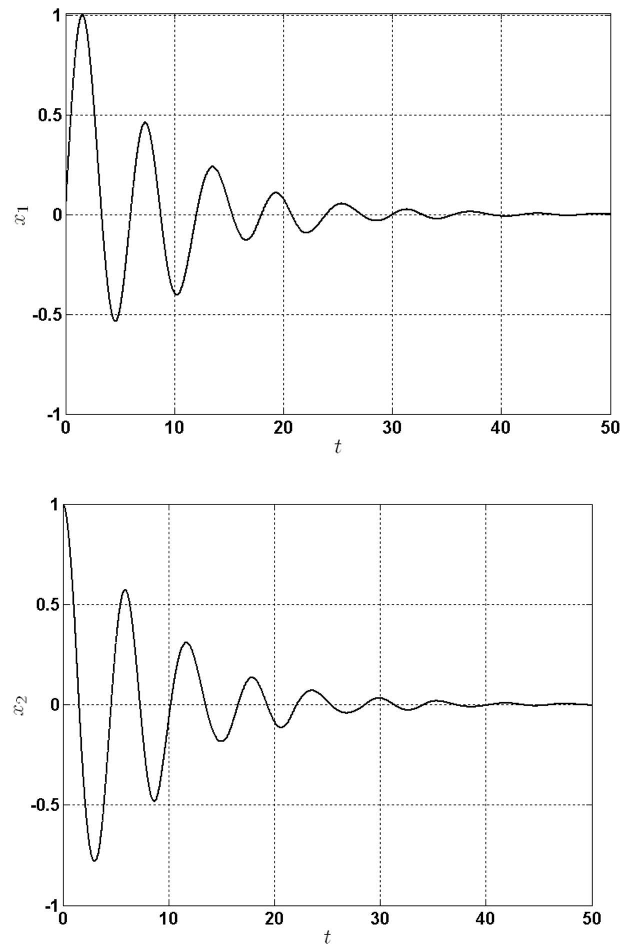

Fig. 1. Evolution of the system states

Solving the Sylvester's equation (10) for

$$R = \begin{pmatrix} -44.5200 & 46.0040\\ 4.4520 & -14.8400\end{pmatrix}$$

leads to

$$S = 10^3\begin{pmatrix} -0.0592 & 0.0958\\ -0.4526 & 1.5394\end{pmatrix}, \quad \tilde{L} = \begin{pmatrix} 0.9994\\ -0.0016\end{pmatrix}.$$

Hence we have

$$\tilde{A} = \begin{pmatrix} 14.6857 & -49.7339\\ 4.3566 & -14.6857\end{pmatrix}, \tilde{B} = 10^{-3}\begin{pmatrix} 2.0026\\ 1.2384\end{pmatrix},$$

$$\tilde{C} = \begin{pmatrix} -59.2393 & 95.7922\end{pmatrix}.$$

Finally, using Sedumi-1.3 for MATLAB we solve LMI system (16) for $\alpha = \beta = \gamma = 0.2$ and obtain

$$K = \begin{pmatrix} 137.1771 & -469.0443\end{pmatrix}.$$

For an interval observer design it is assumed that

$$x(0) \in \Omega := \{x \in \mathbb{R}^2 : |x_i| \le 1, i = 1, 2\}.$$

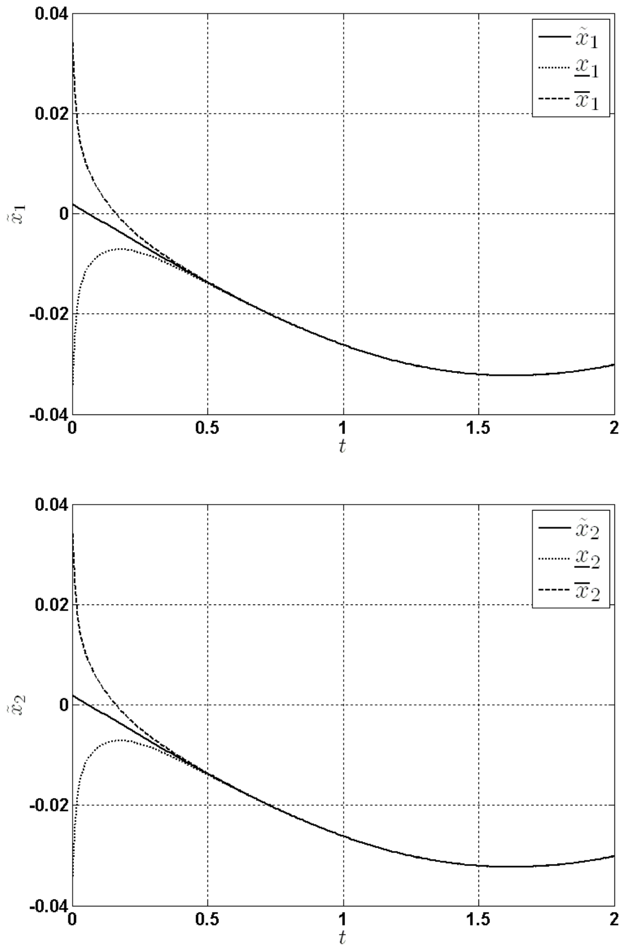

Fig. 2. The real and observed states of the auxiliary vector $\tilde{x}$

Then due to relation $\tilde{x}(0) = S^{-1}x(0)$ the initial conditions for the interval observer (8) can be selected as

$$\overline{x}(0) = |S^{-1}| \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad \underline{x}(0) = -\overline{x}(0),$$

where the modulus of the matrix is understood componentwise.

The figure 1 depicts the results of numerical simulations for the system (1) with the control (15) with the delay $h(t) = 1 + 0.5(1 - \text{sign}(\cos(0.5t)))$ the following initial conditions: $x(0) = (0, 1)^T$ and $v(t) = 0$.

The evolution of the observation process for the auxiliary state vector $\tilde{x} = S^{-1}x$ is shown in the figure 2.

B. Double integrator

The adaptive control scheme presented in [6] also admits unknown time varying input delay, but it is applicable only for a chain of integrators. In order to compare our control algorithm with the one presented in [6] we consider the output control problem for double integrator, i.e. $n = 2, k = m = 1$ and

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, C = \begin{pmatrix} 1 & 0 \end{pmatrix}$$

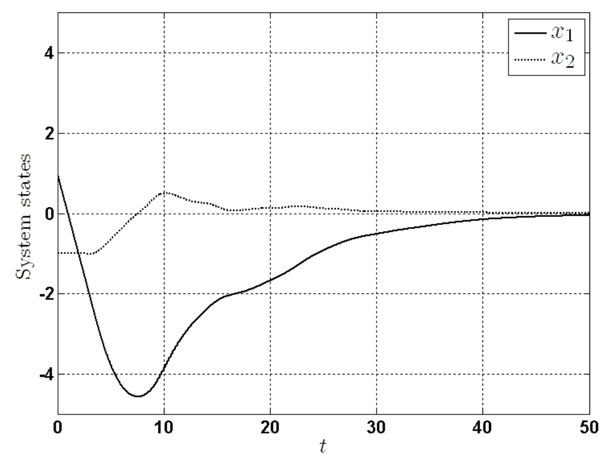

Fig. 3. Evolution of states for controlled double integrator: the case i).

Solving the Sylvester's equation (10) for

$$R = \begin{pmatrix} -3.0000 & 2.3200 \\ 0.2700 & -0.4100 \end{pmatrix}$$

we derive

$$S = \begin{pmatrix} -3.6234 & -0.2599 \\ -1.5558 & -9.1860 \end{pmatrix}, \tilde{L} = \begin{pmatrix} 0.9479 \\ -0.0948 \end{pmatrix}$$

and

$$\tilde{A} = \begin{pmatrix} 0.4346 & 2.5663 \\ -0.0736 & -0.4346 \end{pmatrix}, \tilde{B} = \begin{pmatrix} 0.0079 \\ -0.1102 \end{pmatrix},$$

$$\tilde{C} = \begin{pmatrix} -3.6234 & -0.2599 \end{pmatrix}.$$

The numerical simulations have been done for the same time delay as in [6] :
i) $h(t) = 2\sin(t) + 2$;
ii) $h(t) = 3 + \cos(100t)$;
and for the same initial conditions:

$$x(0) = (1, -1)^T, \quad v(t) = 0, t \in [-\overline{h}, 0).$$

In the case i) we have $\underline{h} = 0$, $\overline{h} = 4$. Solving the LMI system (16) for $\alpha = \beta = \gamma = 0.2$ gives

$$K = \begin{pmatrix} 0.7889 & 3.7978 \end{pmatrix}.$$

For the case ii) the estimates of the delay are $\underline{h} = 2$ and $\overline{h} = 4$. The corresponding vector of feedback gains obtained by the LMI system (16) is the following

$$K = \begin{pmatrix} 1.0747 & 4.5060 \end{pmatrix}.$$

The simulations results for control of double integrator are presented in the figures 3 and 4. They show that the control algorithm based on the interval predictor technique provides faster convergence rate of the system states to the origin comparing with the adaptive scheme presented in [6]. Moreover, in contrast to the adaptive algorithm it shows a better dumping during the transitory motion.

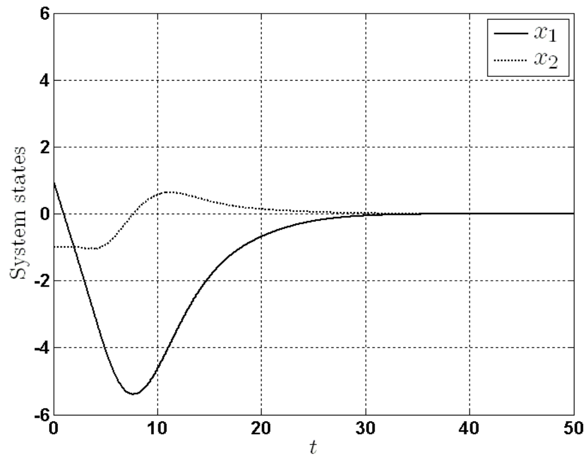

Fig. 4. Evolution of states for controlled double integrator: the case ii).

VII. CONCLUSIONS

In the paper an output-based predictor feedback control algorithm is presented for exponential stabilization of linear system with unknown time-varying input delay using interval predictor and interval observer technique. The procedure of the output feedback design requires the solving of the Sylvester's equation for observer design and finding solution of the LMI system for adjusting of the feedback control gains. The stability analysis of closed-loop system is based on the method of Lyapunov-Krasovskii functionals.

The main results are presented for linear systems with uncertain input delay. However, they can be extended to the case of state and/or output delays and other types of system uncertainties and disturbances. In order to provide more constructive scheme for output feedback design it is necessary to reformulate the procedure for selection of the parameters of the interval observer in a LMI form. The LMI restrictions to the controller and observer parameters will allow us to develop the effective computational scheme for the robust output control design for systems with unknown input delay, measurement noises and exogenous disturbances. Moreover, an optimal selection of the parameters for reduction of the disturbances and noises effects can be provided using Attractive Ellipsoids Method [20] and Semidefinite programming technique.

All these problems are subjected for future researches.

REFERENCES

[1] Z. Artstein. Linear systems with delayed controls: A reduction. *IEEE Transactions on Automatic Control*, 27(4):869–879, 1982.

[2] R. H. Bartels and G. W. Stewart. Solution of the matrix equation AX + XB = C. *Communications of the ACM*, 15:820–826, 1972.

[3] N. Bekiaris-Liberis and M. Krstic. Compensation of time-varying input and state delays for nonlinear systems. *Journal of Dynamic Systems, Measurement, and Control*, 134(1):paper 011009, 2012.

[4] L. Belkoura, J.-P. Richard, and M. Fliess. Parameters estimation of systems with delayed and structured entries. *Automatica*, 45(5):1117–1125, 2009.

[5] D. Bresch-Pietri and M. Krstic. Adaptive trajectory tracking despite unknown input delay and plant parameters. *Automatica*, 45(9):2074–2081, 2009.

[6] H.-L. Choi and J.-T. Lim. Output feedback regulation of a chain of integrators with an unknown time-varying delay in the input. *IEEE Transactions on Automatic Control*, 55(1):263–268, 2010.

[7] D. Efimov, T. Raissi, and A. Zolghadri. Control of nonlinear and LPV systems: interval observer-based framework. *IEEE Transactions on Automatic Control*, 57:to appear, 2012.

[8] L. Farina and S. Rinaldi. *Positive Linear Systems: Theory and Applications*. New York: Wiley Interscience, 2000.

[9] E. Fridman. Descriptor discretized lyapunov functional method: Analysis and design. *IEEE Transactions on Automatic Control*, 51(5):890–897, 2006.

[10] E. Fridman. A refined input delay approach to sampled-data control. *Autom*, 46(2):421–427, 2010.

[11] E. Fridman, A. Seuret, and J.-P. Richard. Robust sampled-data stabilization of linear systems: an input delay approach. *Automatica*, 40(8):1441–1446, 2004.

[12] J.L. Gouze, A. Rapaport, and M.Z. Hadj-Sadok. Interval observers for uncertain biological systems. *Ecological Modeling*, 133(1):46–56, 2000.

[13] K. Gu, V. Kharitonov, and J. Chen. *Stability of Time-Delay Systems*. Boston, MA: Birkhauser, 2003.

[14] M. Jankovic. Recursive predictor design for state and output feedback controllers for linear time delay systems. *Automatica*, 46(3):510–517, 2010.

[15] M. Krstic. *Delay Compensation for Nonlinear, Adaptive, and PDE Systems*. Birkhauser Boston, 2009.

[16] A. Kruszewski, W. J. Jiang, E. Fridman, J.-P. Richard, and A. Toguyeni. A switched system approach to exponential stabilization through communication network. *IEEE Transactions on Control Systems Technology*, 20(4):887–900, 2012.

[17] F. Mazenc and O. Bernard. Interval observers for linear time-invariant systems with disturbances. *Automatica*, 47(1):140–147, 2011.

[18] F. Mazenc, S. Niculescu, and M. Krstic. Lyapunov-Krasovskii functionals and application to input delay compensation for linear time-invariant systems. *Automatica*, 48(7):1317–1323, 2012.

[19] F. Mazenc, S.-I. Niculescu, and O. Bernard. Exponentially stable interval observers for linear systems with delay. *SIAM Journal of Control and Optimization*, 50:285–315, 2012.

[20] A. Polyakov. Minimization of disturbances effects in time delay predictor-based sliding mode control systems. *Journal of The Franklin Institute*, 349(4):1380–1396, 2012.

[21] T. Raissi, D. Efimov, and A. Zolghadri. Interval state estimation for a class of nonlinear systems. *IEEE Transactions on Automatic Control*, 57(1):260–265, 2012.

[22] J.-P. Richard. Time-delay systems: An overview of some recent advances and open problems. *Automatica*, 39(10):1667–1694, 2003.

[23] A. Seuret, T. Floquet, J.-P. Richard, and S. K. Spurgeon. A sliding mode observer for linear systems with unknown time varying delay. In *American Control Conference*, pages 4558–4563, 2007.

[24] O. J. M. Smith. A controller to overcome dead time. *Journal of Instrument Society of America*, 6:28–33, 1959.

[25] A. Teel, D. Nesic, and P. V. Kokotovic. A note on input-to-state stability of sampled-data nonlinear systems. In *Proceedings of the IEEE Conference on Decision and Control*, pages 2473–2478, 1998.

[26] K. Watanabe. Finite spectrum assignment and observer for multivariable systems with commensurate delays. *IEEE Transactions on Automatic Control*, 31(6):543–550, 1986.

[27] E. Witrant, C. Canudas-de Wit, D. Georges, and M. Alamir. Remote stabilization via communication networks with a distributed control law. *IEEE Transactions on Automatic Control*, 52(8):1480–1485, 2007.

[28] D. Yue and Q.-L. Han. Delayed feedback control of uncertain systems with time-varying input delay. *Automatica*, 41(2):233–240, 2005.